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STRATEGIC BRAND PORTFOLIO MANAGEMENT

In the past twenty years or so, three approaches to brand portfolio management strategies have emerged. The first approach is marketing. This approach is associated with building a corporate brand portfolio. The goal is to increase diversified cash flows by entering new market segments. The second approach is related to the competitive strategy of the enterprise. A false portfolio of intellectual property applications is being created. Competitors are expected to spend resources in retaliation. The third approach is the formation of a dynamic strategy for investment portfolio management. Due to the complex structure of the modern global financial market, the heterogeneous structure of available financial instruments and traders using different approaches and time horizons, forecasts, as a rule, require a large number of observations, work poorly in the vicinity of bifurcations and do not have a computer model that could build forecasts in real time. In such structures, slow diffusion-type processes with the phenomenon of memory arise, that is, non-Markov processes. Moreover, such structures can have fractal properties. In this work, it seems to us, the first step has been taken to build a "synthetic" model of dynamic asset portfolio management. By analyzing the data available in the scientific literature, a mathematical model of strategic brand portfolio management is proposed. In view of the above, the model has the form of a differential equation in fractional derivatives. In connection with the risk analysis, two models of fractional entropy are also considered - fractional Kolmogorov-Sinai entropy and fractional Shannon entropy.

Ключові слова: brand portfolio; fractal environment; long-range system; fractional differential equation; fractional entropy.

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СТРАТЕГІЧНЕ УПРАВЛІННЯ ПОРТФЕЛЕМ БРЕНДІВ

За останні двадцять років намітилися три підходи до стратегій управління портфелем брендів. Перший підхід - маркетинговий. Цей підхід пов'язаний із побудовою корпоративного портфеля брендів. Мета - збільшення диверсифікованих грошових потоків завдяки виходу на нові сегменти ринків. Другий підхід пов'язаний із конкурентною стратегією підприємства. Створюється хибний портфель заявок на об'єкти інтелектуальної власності. Передбачається, що конкуренти витратять ресурси на дії у відповідь. Третій підхід – формування динамічної стратегії інвестиційного управління портфелем активів. У силу складного устрою сучасного глобального фінансового ринку, неоднорідної структури наявних фінансових інструментів і трейдерів, що використовують різні підходи часові горизонти, прогнози, як правило, вимагають великої кількості спостережень, погано працюють на околицях біфуркацій і не мають комп'ютерної моделі, яка могла б будувати прогнози у режимі реального часу. У цих структурах виникають повільні процеси дифузійного типу з феноменом пам'яті, тобто немарківські процеси. Крім того, такі структури можуть мати фрактальні властивості. У роботі зроблено, як нам здається, перший крок по побудові «синтетичної» моделі динамічного управління портфелем активів. За допомогою аналізу наявних у науковій літературі даних запропоновано математичну модель стратегічного управління портфелем брендів. У силу сказаного вище, модель має вигляд диференціального рівняння у дробових похідних. У зв'язку з аналізом ризиків розглянуті також дві моделі дробової ентропії - дробова ентропія Колмогорова-Синая та дробова ентропія Шеннона.

Ключевые слова: портфель брендів; фрактальне середовище; феномен пам’яті; диференційне рівняння у дробових похідних; дробова ентропія.

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СТРАТЕГИЧЕСКОЕ УПРАВЛЕНИЕ ПОРТФЕЛЕМ БРЕНДОВ

В последние примерно двадцать лет наметились три подхода к стратегиям управления портфелем брендов. Первый подход – маркетинговый. Этот подход связан с построением корпоративного портфеля брендов. Цель - увеличение диверсифицированных денежных потоков благодаря выходу на новые сегменты рынков. Второй подход связан с конкурентной стратегией предприятия. Создается ложный портфель заявок на объекты интеллектуальной собственности. Предполагается, что конкуренты потратят ресурсы на ответные действия. Третий подход – формирование динамической стратегии инвестиционного управления портфелем активов. В силу сложного устройства современного глобального финансового рынка, неоднородной структуры имеющихся финансовых инструментов и трейдеров, использующих разные подходы и временные горизонты, прогнозы, как правило, требуют большого количества наблюдений, плохо работают в окрестностях бифуркаций и не имеют компьютерной модели, которая могла бы строить прогнозы в режиме реального времени. В таких структурах возникают медленные процессы диффузионного типа с феноменом памяти, то есть немарковские процессы. Кроме того, такие структуры могут иметь фрактальные свойства. В работе сделан, как нам кажется, первый шаг по построению «синтетической» модели динамического управления портфелем активов. С помощью анализа имеющихся в научной литературе данных предложена математическая модель стратегического управления портфелем брендов. В силу сказанного выше, модель имеет вид дифференциального уравнения в дробных производных. В связи с анализом рисков рассмотрены также две модели дробной энтропии – дробная энтропия Колмогорова-Синая и дробная энтропия Шеннона.

Keywords: портфель брендов; фрактальная среда; феномен памяти; дифференциальное уравнение в дробных производных; дробная энтропия.

Introduction. Modern economic practice is characterized by the presence of structurally unstable markets included as nodes in the real-time network of the global "new economy". In this regard, the analytical component begins to play a key role in the management of any business: the function of providing management with a decision-making system in a rapidly changing external environment.

Competitive strategy (CS) defines how the firm intends to compete in its field, the mission of the firm and the means of its implementation. The most important aspect of CS is the alignment between the firm's mission and the means by which it is achieved. The most powerful are the resources that form competitive advantages. One

of the most universal and effective tools for protecting competitive advantage and a powerful offensive factor in the CS of innovative business are objects of intellectual property (IP). They are objects of exclusive rights and hinder the desire of competitors to eliminate the previously achieved innovative advantage.

In 1991, Aaker's fundamental work “Capitalizing on the value of a brand name” appeared; soon began the activity of the company "Interbrand" to assess the market value of brands. Today there are approximately 60 global brands worth over a billion dollars. For example, the Coca-cola brand is worth $84 billion and represents 59% of the company's value; the Nokia brand is worth $21 billion and accounts for 44% of the company's value; the Nescafe brand is worth $18 billion and represents 23% of the company's value. The valuation techniques used are based on the allocation of the revenue stream from the brand and its indirect capitalization. The discount rate is determined by the cumulative construction method.

Under dynamic portfolio management, we will change its structure in real time to improve the investment properties of the portfolio. In today's global economy, more and more attention is paid to the formation of brand portfolios. It is about how brands are related to each other, how they interact, what functions they perform. In principle, three types of brand portfolios can be distinguished: "corporate portfolio", "portfolio of competitive strategies" and "investment portfolio". In this regard, the key role in the management of any business begins to play the provision of synergistic effects from the formation of brand portfolios. This method gives a rather rough estimate and does not allow building dynamic strategies for managing even one brand, especially in the face of possible chaotic behavior of the securities markets. in the modern economy, more and more attention is paid to the formation of brand portfolios. It is about how brands are connected, how they interact, what functions they perform. In this regard, ensuring synergistic effects from the formation of brand portfolios begins to play a key role in the management of any business. The main thing here is to ensure the adequacy of your competitive strategy, its relevance in a rapidly changing environment.

Analysis of recent research. Currently, the most actively discussed issues of strategies for managing corporate portfolios of brands. As goal-setting, the tasks of synergy within the portfolio are studied. A fairly complete picture of the theoretical aspects of these problems can be obtained from the works [1-7].

The second type of brand portfolios is closely related to competitive strategies. Competitive strategy (CS) determines how the firm intends to compete in its area, the mission of the firm and the means of its implementation. The most powerful resources are those that create competitive advantages. Theoretically, one of the most versatile and effective tools for protecting a competitive advantage and a powerful offensive factor in the CS of an innovative business are objects of intellectual property (OIP).

They are objects of exclusive rights and prevent competitors from striving to eliminate the previously achieved innovative advantage. On the other hand, when filing an application for registration of an OIP, a company shows the direction of its development. Therefore, the practice of filing false applications has been developing recently. For obvious reasons, works related to the practical application of competitive strategies are not published.

The third type of brand portfolio is important for portfolio investors. In 1991, Aaker's fundamental work “Capitalizing on the value of a brand name” appeared; soon began the activity of the firm "Interbrand" to assess the market value of brands. Today there are about 60 global brands worth over a billion dollars.

The valuation techniques used are based on the separation of the brand income stream and its indirect capitalization. The discount rate is determined by the cumulative construction method. This method gives a rather rough estimate and does not allow building dynamic management strategies for even one brand, especially in conditions of the possible chaotic behavior of the securities markets.

In general, it should be noted that in all the cases considered above, there are no adequate mathematical models of strategic brand portfolio management.

Formulation of the problem. The classical models of all portfolio theories require the fulfillment of certain hypotheses for the asset market. The 2008 crisis showed that these hypotheses have not been fulfilled for a long time. After that, almost all known mathematical formalizations of the market functioning began to use one or another model of diffusion-type processes in a stationary environment.

Basically, this was achieved by the transition to the study of the series of returns, and not rates. However, such processes "have no memory." By the way, this property is one of the main postulates of technical analysis. On the other hand, data on rates of past periods is used to predict rates. They are implicitly trying to remove this contradiction by different methods in different approaches to forecasting. In addition, a specific market for specific assets is still being considered (and not really existing now global).

Globalization is characterized by the presence of structurally unstable markets included as nodes in the real-time network of the world economy. It follows from the theory of dynamical systems that a model describing such a specific system must contain irreparable chaos regions. Therefore, until quite recently, the methods of the theory of chaotic dynamics and the theory of bifurcations were an adequate mathematical apparatus. In the last ten years, the structure of the market has become even more complex.

Currently, we should consider the global market for heterogeneous assets with an almost infinite number of agents with different types of behavior and different investment horizons. Note that the effects associated with the topological structures of this market have not yet been investigated. The real dynamics of the rate change looks like a fractal process in time; therefore, infraslow processes of a diffusion type arise. The rate charts under

the influence of traders depend not only on the "now" state, but are determined by the state of the entire chart.

Such presence of long-range action can lead to the appearance of the phenomenon of memory, while fractional derivatives appear in the equations describing the dynamics. These considerations lead us to the use of probabilistic interpretations of fractional derivatives [8,9] and further to equations of diffusion type in fractional derivatives. The fractional time derivative is understood in the sense of Grunwald – Letnikov, and the spatial fractional derivative - in the sense of Marshaud [10].

Mathematical model.

Mathematical model of dynamic portfolio management of corporate brands.

The strategy of managing a portfolio of corporate brands is relevant when planning business growth and increasing its efficiency. At the stage of economic growth, it is generally required to plan for sustainable growth and the development of new markets. At the stage of economic decline, it is mainly required to plan an increase in business efficiency. Promotion of corporate brands usually takes place in markets with high liquidity and a large number of participants with different investment horizons.

Therefore, it is appropriate to define risks using Shannon's fractional information entropy. Taking all these considerations into account, we will write a dynamic model for assessing a portfolio of corporate brands:

$$ {}^{GL}_{\gamma}DX = -H(X)^{M}_{\delta}\,DX + {}^{M}_{\gamma}DX \ . $$

Here X is the expected value of the corporate portfolio, $H(X)$ is the Shannon fractional entropy, γ is the Hurst exponent, ${}^{GL}_{\gamma}DX$ is the fractional Grunwald-Letnikov derivative of order γ, ${}^{M}_{\delta}DX$ is the fractional Marshaud derivative of order δ, δ is the Hausdorff-Besicovitch dimension.

Mathematical model of dynamic portfolio management of competitive strategies for IT businesses.

It makes sense for IT businesses to define the value of a competitive strategy as the value of a firm's share in a rapidly growing market with rapidly changing new risks. Therefore, we consider only this case as a simpler one. Methods of the theory of equations in fractional derivatives are not needed to describe such a process. There are enough dynamic models with cycle bifurcations and stochastic modes.

It follows from the general theory that almost any process with real-time changing asset values is a K-system and has an area of chaos (which is currently observed in practice). Obviously, the Kolmogorov - Sinai entropy of the dynamics of the NASDAQ composite stock index should be considered a measure of systematic risk in the tasks of assessing the competitive strategy of IT businesses.

When building models of venture investment in a specific project for a specific investor, it is also necessary to model non-systematic risks of a specific IT business. The function of a non-systematic risk component for a specific IT business can be determined from a dynamic system.

$$ \dot{R} = -bR(t) + \sigma(N)R(t)S(t) \ , $$

where $S(t)$ is the cost of the company's competitive strategy, $b \in R, b \geq 0$ is a parameter, $\sigma(N)$ is the standard deviation of the company's profitability in the stock index base NASDAQ composite.

All strategic decisions made today will be implemented in the future. The effectiveness of the decisions made directly depends on the degree of completeness and quality of information resources and models for assessing, making and maintaining decisions. Competitive intelligence provides the optimal selection of information resources. In the West, 99,9% of such resources are obtained from open sources, about 70% of which are professional on-line databases.

A feature of IT businesses is that potential investors determine their value based on the company's potential share in a structurally unstable market. Therefore, in order to develop a competitive strategy based on competitive intelligence data under conditions of "stochastically information-behavioral" risks, adequate dynamic models are required that depend on parameters and have bifurcations of cycles and regions of chaos.

The cost of a competitive strategy for a specific IT business is determined from a dynamic system:

$$ \dot{S} = h(N)S(t)[1 - \int_{-\infty}^{0} S(t+s)Q(-s)ds] - \alpha R(t)S(t) \ , $$

$$ \dot{R} = -bR(t) + \sigma(N)R(t)S(t) \ , $$

where $\alpha \in R, \alpha \geq 0$ is the parameter, and the function, $Q(t) = [t / \mathrm{var}(T)]\exp(-t / \sigma(T))$, $T = h(D)$ is the Kolmogorov entropy - Sinai dynamics of the cycle period of the NASDAQ composite stock index.

This two-parameter model has two bifurcation points: the first corresponds to a soft loss of stability of equilibrium with the formation of self-oscillations, the second corresponds to a soft loss of stability of the cycle and a transition to a stochastic regime (such as the Feigenbaum effect).

Mathematical model of dynamic portfolio management of investors brands.

In this case, when constructing portfolio branding models, the Kolmogorov - Sinai entropy h (I) of the dynamics of the corresponding stock index I should be considered a measure of systematic risk in the problems of assessing brand portfolios.

The measure of the non-systematic component of risk in the problems of assessing brand portfolios should be considered the topological entropy defined for the brand space, which should be constructed in such a way that the Hausdorff - Besicovich dimension of the fractal set coincides with the metric entropy. The corresponding conditions are contained in the theorem of Besicovitch and Eggleston. These conditions, in fact, establish the relationship between the dynamic and fractal aspects of the chaotic behavior of the securities markets.

The function R_i of the non-systematic risk component for a new specific brand in the portfolio can be determined from the dynamic system

$$ {}^{GL}_{\gamma}DR_i = \beta_i F^T - \lambda_i R_i , $$

where ${}^{GL}_{\gamma}D$ is the Grunwald-Letnikov derivative, $\psi = col(\psi_1, ..., \psi_m)$, ψ_j is the value of the *j*-th new brand, are parameters (which are determined by methods of expert assessments and / or methods of technical analysis), *F* is a matrix that describes the "splitting" of the portfolio.

In this case, the value of a brand portfolio is determined from a dynamic system:

$$ V^{-1}\,{}^{GL}_{\gamma}\psi = \left({}^{M}_{\delta}grad, H\,{}^{M}_{\delta}\psi \right) + \left((1-\beta)\chi F^T - A \right)\psi $$

$$ + \sum_{i=1}^{n} \lambda_i \chi_i R_i $$

$$ {}^{GL}_{\gamma}R_i = \beta_i F^T \psi - \lambda_i R_i $$

Here $V = diag(v_1, ..., v_m)$, ${}^{M}_{\delta}grad$ fractional Marshaud gradient, v_j the rate of change in the value of the j-th new brand, the matrices A, χ, H describe, respectively, the game matrix, "emission spectrum" and "Shannon fractional diffusion" for different classes of brands. Management of "emission" of new brands and brand portfolio is carried out on the basis of mixed strategies - game solutions *A.*

Results of work. Mathematical models of dynamic management of brand portfolios were obtained in the form of differential equations in fractional derivatives.

Prospects for further research. In the future, it is quite possible to improve the proposed models of dynamic portfolio management by developing correct methods for selecting the factors most affecting the forecast.

Conclusions. This work is an introduction to the creation of software implementations of predictive dynamic models that can work in real time.

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